Code of examination paper

Exercises of National Exam in Mathematics 02.05.2006

Part I

1. You have to solve 7 problems.
2. It is not needed to rewrite the texts of the problems on the solutions sheet
3. The solution of each problem must be written at the place foreseen for it.
4. If the space foreseen for you solution is not sufficient, you should ask an extra sheet from the examination commission and continue the solution. Write your remark about continuation at the end of place foreseen for the solution.
5. Before you hand your work to the examination commission, put between it the sheet with texts of problems provided with your code and the extra sheet (if you have one) with your code. Please don't interpose your rough copy.

1. (5 points) Given the expression $\sqrt{\left(\dfrac{b}{a}\right)^{-2}} \cdot \left(\dfrac{a-b}{\sqrt{a}-\sqrt{b}} - \dfrac{a-b}{\sqrt{a}+\sqrt{b}}\right)$, where $a > b > 0$.

   1) Simplify the expression.

   2) Find the value of the expression for $a = 2^{0}$ and $b = 3^{-2}$.

2. (5 points) A chemical needed for the experiment is in ampules in two boxes. There are 16 ampuls in one box, at which the contents of two of them is expired, and 19 ampules in another box with 4 expired of them. A pupil takes at random from the randomly chosen box an ampule. What is the probability that the pupil will take:

   1) an ampule from the box with the smaller number of expired ampuls,   1 point

   2) an ampule which is not expired.   4 points

3. (5 points) A girl wants to swim across the river. The speed of the flow is $0{,}3\,\dfrac{m}{s}$. She can swim in slack water at the rate of $1{,}5\,\dfrac{m}{s}$. What must be the swimming angle with regard to the river-bank in order to reach the opposite bank at the point opposite to this one where she had gone into water (see Fig.).

direction of flow
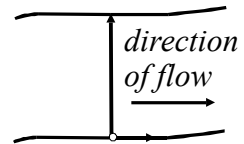
4. (5 points) Find the values of a for which the equation $\cos x = 5a - 2$ has a solution that belongs to interval $[0; \dfrac{\pi}{2}]$.

5. (10 points) Given the functions $f(x) = \dfrac{1}{3}x^{3} - x$ and $g(x) = -x + 1$.

   1) Find the zeros, maximum and minimum of the function $y = f(x)$.   6 points

   2) Sketch the graphs of the functions $y = f(x)$ and $y = g(x)$ on the same co-ordinate axes on the interval $[-3; 3]$.   2 points

   3) Write out the common interval where both the functions are decreasing.   2 points

6. (10 points) A company spends $y = x^{3} + 2000x + 40000$ dollars to manufacture x units (e.g. TV sets). The company sells its products \$9500 apiece.

   1) Calculate the profit that the company will get having produced and sold 10 units.   3 points

   2) How many units must the company produce and sell to make the maximal profit?   7 points

7. (10 points) The riverside cornfields were over flooded, and the yield was destroyed. The insurance company has paid to the owners \$1000 compensation per hectar. What was the compensation to an owner whose field is a quadrangle (see Fig.) with two perpendicular sides of 0,5 km and 1,5 km and with an interior angle of $120°$ by the shorter side and an interior angle of $45°$ by the longer side. Give the answer with accuracy of \$$10^{3}$.

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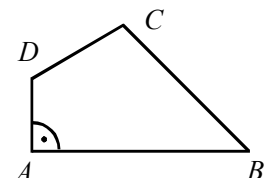
1st variant

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Part II

Two exercises 8. and 9. have to be solved and additionally either exercise 10. or 11.
Only the solutions of 3 (two 15-point and one 20-point) exercises will be evaluated.
Please write serial numbers of the exercises presented for evaluation

      before the respective solutions and
      in the respective squares on the cover of the examination paper.

8. (15 points) The sides of an isosceles triangle lie on the lines $2x + 3y - 12 = 0$ and
$3x + 2y - 12 = 0$. The midpoint of the base is $K(-0,6; 5,4)$. Find

1) the co-ordinates of the intersection point of the sides,	4 points
2) the equation of the line, on which the base of the triangle lies,	7 points
3) the length of the altitude (give answer in irrational form).	4 points

9. (15 points) Given the functions $f(x) = \ln(4x)$ and $g(x) = -\ln x$.

1) Find the co-ordinates of the intersection points of the graphs of the given functions.	4 points
2) Present the expression of the $f(x)$ by sum.	1 point
3) Compile the equations of the tangent lines to the graphs of both functions at the intersection point.	5 points
4) Sketch on the same co-ordinate axes the graphs of the functions $y = f(x)$ and $y = g(x)$, and the tangent lines found in 3).	5 points

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10. (20 points) The cross-section of the pipes of the cabletrass is a circle
with the diameter *d*. The pipes are laid on a fork stand so that there is
one pipe on the first layer and every following layer the two pipes
contact one pipe on the previous layer (see Fig.). There are *m* pipes
on the uppermost layer.

1) Express

      a) a width *l* of the uppermost layer,

      b) the height *h* of the pile of pipes,

      c) the number of pipes in the pile.

2) Calculate the height of the pile of pipes if the diameter of the
cross-section of a pipe is 12 cm and there are 21 pipes on the
uppermost layer.
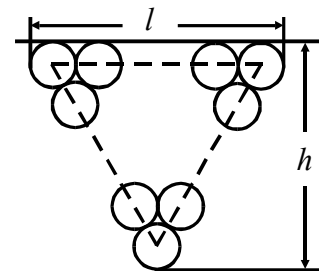
11. (20 points) A trough (see Fig.) has the ends of the shape of isosceles trapezoids, one base of
which is by 30% longer then another and which are perpendicular to the bottom. The sides of the
trough are rectangle shaped. The bottom is a rectangle with the width *a*.

The depth of the trough is *h*, and the depth of the
water in it is 0,5 *h*. The trough is tilted aside as
far as the (rectangle shaped) opposite wall fully
comes out of water. Make sense if some water
will brim over.