Code of examination paper

1st variant

Exercises of National Exam in Mathematics 18.05.2007

Part I

1. You have to solve 6 problems.
2. It is not needed to rewrite the texts of the problems on the solutions sheet
3. The solution of each problem must be written at the pace foreseen for it.
4. If the space foreseen for you solution is not sufficient, you should ask an extra sheet from the examination commission and continue the solution. Write your remark about continuation at the end of place foreseen for the solution.
5. Before you hand you work to the examination commission, put between it the sheet with texts of problems provided with your code and the extra sheet (if you have one) with your code. Please don't interpose your rough copy.

1. (5 points) Given the expression $\dfrac{1+5x}{x^{-2}\cdot\left(25x^2-x^0\right)}$, where $x \neq 0$ and $x \neq \pm\dfrac{1}{5}$.

1) Simplify the expression.

2) Calculate the value of the expression for $x = 2^{\frac{3}{2}}$ with accuracy 10^{-2}.

2. (5 points) A box contains 10 yellow and 6 green balls in the box. What is the probability of

1) taking at random from the box a green ball; — 1 point
2) taking together at random from the box two green balls. — 4 points

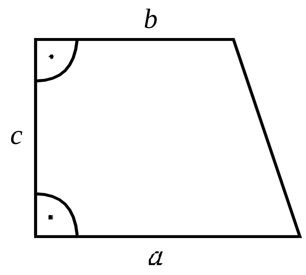
3. (10 points) Given the function $y = x^3 - 5x^2 + 3x + 7$.

1) Find the intervals of decreasing and increasing of the function. — 6 points
2) Calculate the minimum value of the function on the interval $[-2;4]$. — 2 points

4. (10 points) Given the function $y = 2\sin x$ on the interval $[0;2\pi]$.

1) Find the zeros and the range of the function (variation) — 6 points
2) Sketch the graph of the function. — 2 points
3) Using the graph obtained find — 2 points
   a) the domains of positivity and negativity of the function;
   b) the value of the argument x for which $y < -1$.

5. (10 points) A fountain is situated at the intersection point of diagonals of the right-angled trapezoid from pond



1) Find the distance of the fountain from the bank corresponding to the longer base of the trapezoid if the lengths of the banks corresponding to the bases are a and b ($a > b$) and the length of the bank perpendicular to them is c (see Fig.)
2) Calculate this distance if a = 60 m, b = 40 m and c = 30 m.

6. (5 points) In a cold room where the temperature was 0°C, the radiator was switched on, and the temperature began to rise. In the first hour the temperature had risen to 5°C. The change in temperature for every following hour was a certain multiple k of the temperature change during the preceding hour. At the end of the third hour the temperature in the room was 10°C.

1) Calculate the multiple k.
2) What will be the temperature in the room if the number of hours is unrestrictedly increasing. — 6 points

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Part II

Two exercises 7. and 8. have to be solved and additionally either exercise 9. or 10.
Only the solutions of 3 (two 15-point and one 20-point)exercises will be evaluated.
Please write serial numbers of the exercises presented for evaluation

before the respective solutions and
in the respective squares on the cover of the examination paper.

7. (15 points) The bases of an isosceles trapezoid *ABCD* are parallel to the y – axis, and the x – axis of symmetry of the trapezoid. Given the verities $A(1{,}5;-5{,}5)$ and $\overrightarrow{AD} = (3{,}2;2{,}4)$. Draw the figure.

Find:

1) the area of the trapezoid; 4 points
2) the base angle of the trapezoid; 7 points
3) the equation of the line on which the side *AD* lines. 4 points

8. (15 points) Given the curve $y = x\ln x + 2x$.

1) Find on this curve a point $P(x;y)$ for which the sum of the co-ordinates is the least. 4 points
2) Find a number a for which the line $y = ax - 2$ is a tangent to the given curve. 1 point
Calculate the co-ordinates of this point of tangency.

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9. (20 points) It is known of the cubic function $y = ax^3 + bx^2 + cx + 1$ that in the set all tangents to its graph there is only with the slope 4, and it contacts at the point with $x = -\dfrac{1}{3}$. Additionally, it is known that the cubic function has an extremum at the point $x = -1$. Determine the coefficients *a*, *b* and *c*.

10. (20 points) On the base of a cone there are four spheres of equal size thereby each of them contacts the two of the three left. A fifth sphere of the same size lies on these spheres (see Fig.). Each sphere contacts the lateral surface of the cone. Find the distance from the highest point of the fifth sphere to the base of the cone and the value of the vertex angle in the axial cut of the cone if the radius of the sphere is *r*.